Quantum fluctuations and stability of dipolar fermionic and bosonic systems

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There once was a Born-Infeld theory. . .

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PHYSICAL REVIEW D, VOLUME 65, 065007

Dyons in non-Abelian Born-Infeld theory

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(Received 26 October 2001; published 15 February 2002)

We analyze a non-Abelian extension of the Born-Infeld action for the $SU(2)$ group. In the class of spherically symmetric solutions we find that, in addition to the Gal'tsov-Kerner glueballs, only the analytic dyons have finite energy. The presented analytic and numerical investigation excludes the existence of pure magnetic monopoles of 't Hooft–Polyakov type.

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. . . and some other official stuff over the years

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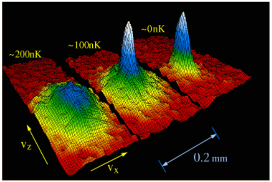
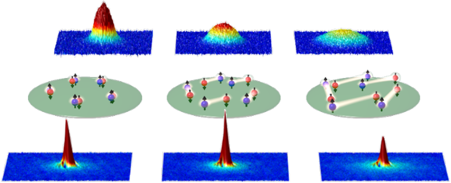
Ultracold quantum gases

- Bose-Einstein condensation
- Quantum degenerate Fermi gases
- Tunability of interactions and geometry
- BEC-BCS crossover
- Optical lattices - simulating solid state physics

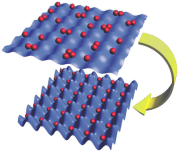
2 D velocity distributions



Science **269**, 198 (1995)



Phys. Rev. Lett. **114**, 230401 (2015)



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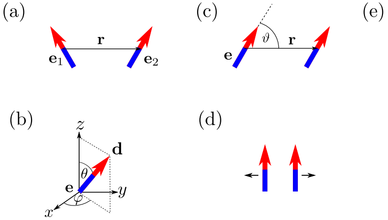
Dipole-dipole interaction (DDI)



- DDI potential: $V_{\mathrm{dd}}(\mathbf{r})=\frac{C_{\mathrm{dd}}}{4\pi}\frac{\mathbf{r}^2-3(\mathbf{e}\cdot\mathbf{r})^2}{|\mathbf{r}|^5}$
- $C_{\mathrm{dd}} = \mu_0 m^2$ for magnetic dipole moment m
  Dipolar atoms: ^{53}Cr, ^{164}Dy, ^{167}Er, ...
- $C_{\mathrm{dd}} = d^2/\varepsilon_0$ for electric dipole moment d
  Dipolar molecules: ^{40}K^{87}Rb, ^{23}Na^{40}K, ...

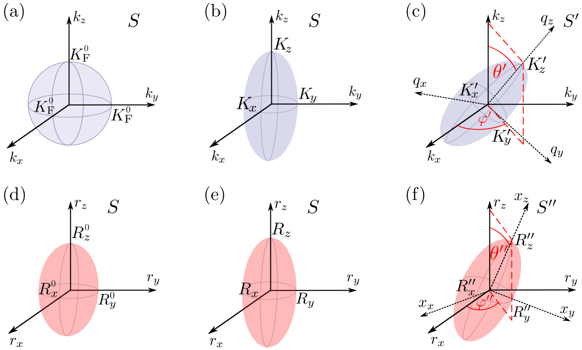
Motivation

- Observation of the Fermi surface (FS) deformation

Science **345**, 1484 (2014)

- Realization of the degenerate Fermi gas of polar molecules

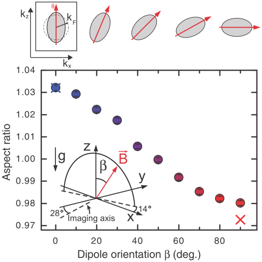
Science **363**, 853 (2019)



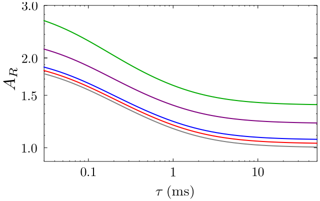
Phys. Rev. A **77**, 061603(R) (2008)

Goals

- Generalization of the Hartree-Fock mean-field theory to describe trapped Fermi gases with tilted dipoles at $T = 0\,\mathrm{K}$
- Extending of the existing theoretical models for dynamics of dipolar fermions for all experimentally relevant regimes



Science **363**, 853 (2019)



Veljić et al., Phys. Rev. A **95**, 053635 (2017)

Ground state

- Second–quantized Hamiltonian for dipolar Fermi gases:

$$\hat{H}=\int d\mathbf{r}\hat{\Psi}^\dagger(\mathbf{r})\left[-\frac{\hbar^2}{2M}\Delta+V_{\rm trap}(\mathbf{r})\right]\hat{\Psi}(\mathbf{r})+\frac{1}{2}\iint d\mathbf{r}d\mathbf{r}'\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r}')V_{\rm dd}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}')\hat{\Psi}(\mathbf{r})$$

- Wigner function:

$$\nu^0(\mathbf{r},\mathbf{k})=\int d\mathbf{s}\, e^{i\mathbf{k}\cdot\mathbf{s}}\left\langle\hat{\Psi}^\dagger\left(\mathbf{r}+\frac{\mathbf{s}}{2}\right)\hat{\Psi}\left(\mathbf{r}-\frac{\mathbf{s}}{2}\right)\right\rangle,\ \text{with } n(\mathbf{r})=\int\frac{d\mathbf{k}}{(2\pi)^3}\nu(\mathbf{r},\mathbf{k})$$

- Total energy of the system in Hartree-Fock approximation:

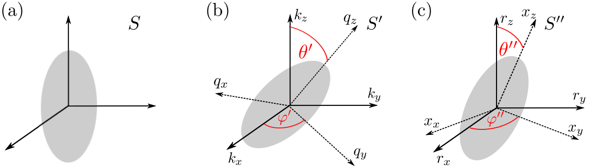
$$E_{\rm kin}=\iint\frac{d\mathbf{r}d\mathbf{k}}{(2\pi)^3}\frac{\hbar^2k^2}{2M}\nu^0(\mathbf{r},\mathbf{k})\,,$$

$$E_{\rm trap}=\iint\frac{d\mathbf{r}d\mathbf{k}}{(2\pi)^3}V_{\rm trap}(\mathbf{r})\nu^0(\mathbf{r},\mathbf{k})\,,$$

$$E_{\rm dd}^{\rm D}=\frac{1}{2}\iiiint\frac{d\mathbf{r}d\mathbf{r}'d\mathbf{k}d\mathbf{k}'}{(2\pi)^6}V_{\rm dd}(\mathbf{r}-\mathbf{r}')\nu^0(\mathbf{r},\mathbf{k})\,\nu^0(\mathbf{r}',\mathbf{k}')\,,$$

$$E_{\rm dd}^{\rm E}=-\frac{1}{2}\iiiint\frac{d\mathbf{r}d\mathbf{r}'d\mathbf{k}d\mathbf{k}'}{(2\pi)^6}V_{\rm dd}(\mathbf{r}')e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}'}\nu^0(\mathbf{r},\mathbf{k})\,\nu^0(\mathbf{r},\mathbf{k}')\,.$$

Wigner function ansatz



Veljić et al., Phys. Rev. Research **1**, 012009 (2019)

- Ansatz for the Wigner function at zero temperature:

$$\nu^0(\mathbf{r},\mathbf{k})=\Theta\left(1-\sum_{i,j} r_i\mathbb{A}_{ij}r_j-\sum_{i,j} k_i\mathbb{B}_{ij}k_j\right)$$

$$\mathbb{A}''=\begin{pmatrix}1/R_x''^2 & 0 & 0\\ 0 & 1/R_y''^2 & 0\\ 0 & 0 & 1/R_z''^2\end{pmatrix} \quad \text{and} \quad \mathbb{B}'=\begin{pmatrix}1/K_x'^2 & 0 & 0\\ 0 & 1/K_y'^2 & 0\\ 0 & 0 & 1/K_z'^2\end{pmatrix},$$

$$\mathbb{R}(\theta,\varphi)=\begin{pmatrix}\cos\theta\cos\varphi & -\sin\varphi & \sin\theta\cos\varphi\\ \cos\theta\sin\varphi & \cos\varphi & \sin\theta\sin\varphi\\ -\sin\theta & 0 & \cos\theta\end{pmatrix}$$

Thomas-Fermi radii and momenta

- Total energy of the system:

$$E_{\rm tot}=\frac{N}{8}\left(\sum_i \frac{\hbar^2 K_i'^2}{2M}+\sum_{i,j}\frac{M\omega_i^2\mathbb{R}_{ij}''^2 R_j''^2}{2}\right)-\frac{6N^2c_0}{R_x''R_y''R_z''}$$
$$\times\left[F_A\left(\frac{R_x''}{R_z''},\frac{R_y''}{R_z''},\theta,\varphi,\theta'',\varphi''\right)-F_A\left(\frac{K_z'}{K_x'},\frac{K_z'}{K_y'},\theta,\varphi,\theta',\varphi'\right)\right]$$

- Generalized anisotropy function:

$$F_A(x,y,\theta,\varphi,\tilde{\theta},\tilde{\phi})=\left(\sum_i \mathbb{R}_{iz}\tilde{\mathbb{R}}_{ix}\right)^2 f\left(\frac{y}{x},\frac{1}{x}\right)+\left(\sum_i \mathbb{R}_{iz}\tilde{\mathbb{R}}_{iy}\right)^2 f\left(\frac{x}{y},\frac{1}{y}\right)+\left(\sum_i \mathbb{R}_{iz}\tilde{\mathbb{R}}_{iz}\right)^2 f(x,y)$$

Veljić et al., Phys. Rev. Research **1**, 012009 (2019)

- Particle number conservation:

$$N=\frac{1}{48}R_x''R_y''R_z''K_x'K_y'K_z'$$

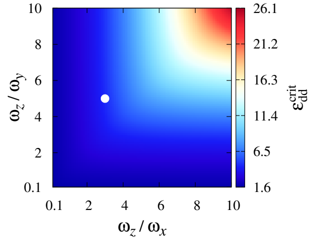
- The minimization of $E_{\rm tot}$ leads to the set of 10 equations for 10 variational parameters $(R_i'',K_i',\theta',\varphi',\theta'',\varphi'')$

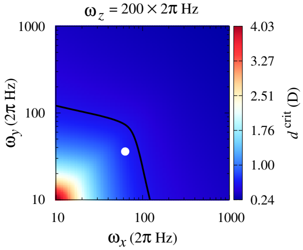
Stability diagrams

- Non-dimensional form of equations: species-independent!
- Relative interaction strength:

$$\varepsilon_{\rm dd} = \frac{d^2}{4\pi\varepsilon_0}\sqrt{\frac{M^3}{\hbar^5}}(\omega_x\omega_y\omega_z N)^{1/6}$$

- Stability only depends on the trap aspect ratios and the dipoles' orientation





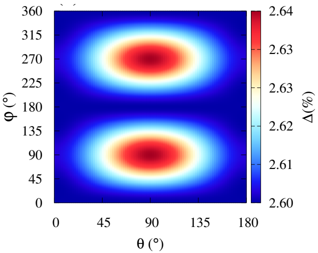
Veljić et al., Phys. Rev. Research **1**, 012009 (2019)

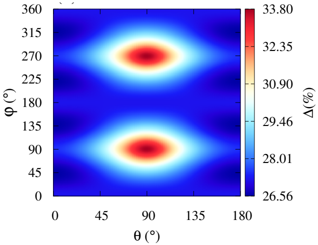
Fermi surface deformation

- Our theory confirms that Fermi surface (FS):
  1. follows the dipoles' orientation $(\theta'=\theta, \varphi'=\varphi)$
  2. remains cylindrically symmetric ellipsoid $(K'_x=K'_y)$
- Fermi surface deformation:

$$\Delta=\frac{K'_z}{K'_x}-1$$

- $N = 6.7 \cdot 10^4$, (579, 91, 611) Hz

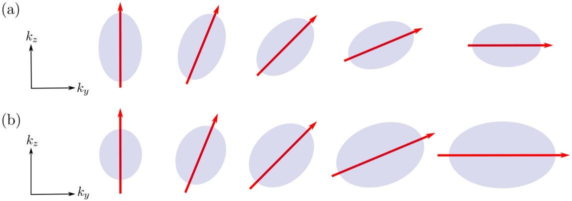




Veljić et al., New J. Phys. **20**, 093016 (2018)

Fermi surface deformation

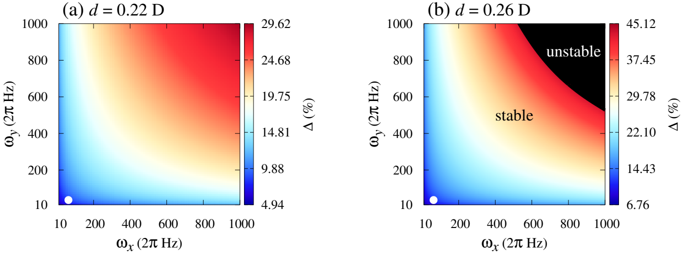
- Dipolar atoms → rigid FS; Polar molecules → soft FS



- FS deformation and its angular distribution can be tuned by changing the trap frequencies and dipoles' orientation
- For stronger DDI we expect increased critical temperature of Cooper pairing, but also a higher degree of tunability

Fermi surface deformation - KRb

- FS deformation is much larger in gases of polar molecules
- $N = 3 \cdot 10^4$, $\omega_z = 2\pi \times 200$ Hz, $\theta = \varphi = 0$
- Even small changes in the dipolar moment strength can significantly affect the systems' stability



Veljić et al., Phys. Rev. Research **1**, 012009 (2019)

Quantum Boltzmann kinetic equation

- Second–quantized Hamiltonian for dipolar Fermi gases:

$$\hat{H}=\int d\mathbf{r}\hat{\Psi}^{\dagger}(\mathbf{r},t)\left[-\frac{\hbar^2}{2M}\Delta+V_{\text{trap}}(\mathbf{r})\right]\hat{\Psi}(\mathbf{r},t)+\frac{1}{2}\int d\mathbf{r}d\mathbf{r}'\hat{\Psi}^{\dagger}(\mathbf{r},t)\hat{\Psi}^{\dagger}(\mathbf{r}',t)V_{\text{dd}}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}',t)\hat{\Psi}(\mathbf{r},t)$$

- Wigner function:

$$\nu(\mathbf{r},\mathbf{k},t)=\int d\mathbf{s}\, e^{i\mathbf{k}\cdot\mathbf{s}}\left\langle\hat{\Psi}^{\dagger}\left(\mathbf{r}+\frac{\mathbf{s}}{2},t\right)\hat{\Psi}\left(\mathbf{r}-\frac{\mathbf{s}}{2},t\right)\right\rangle, \text{ with } n(\mathbf{r},t)=\int\frac{d\mathbf{k}}{(2\pi)^3}\nu(\mathbf{r},\mathbf{k},t)$$

- Dynamics of the system:

$$\frac{\partial\nu(\mathbf{r},\mathbf{k},t)}{\partial t}+\frac{\hbar\mathbf{k}}{M}\nabla_{\mathbf{r}}\nu(\mathbf{r},\mathbf{k},t)+\nabla_{\mathbf{k}}V(\mathbf{r},\mathbf{k},t)\nabla_{\mathbf{r}}\nu(\mathbf{r},\mathbf{k},t)-\nabla_{\mathbf{r}}V(\mathbf{r},\mathbf{k},t)\nabla_{\mathbf{k}}\nu(\mathbf{r},\mathbf{k},t)=I_{\text{coll}}[\nu](\mathbf{r},\mathbf{k},t)$$

- Hartree-Fock mean-field potential:

$$V(\mathbf{r},\mathbf{k},t)=V_{\text{trap}}(\mathbf{r})+\int d\mathbf{r}'V_{\text{dd}}(\mathbf{r}-\mathbf{r}')n(\mathbf{r}',t)-\int\frac{d\mathbf{k}'}{(2\pi\hbar)^3}\tilde{V}_{\text{dd}}(\mathbf{k}-\mathbf{k}')\nu(\mathbf{r},\mathbf{k}',t)$$

- Relaxation-time approximation: $I_{\text{coll}}[f]=-\frac{\nu-\nu^{\text{hy}}}{\tau}$

Ansatz for Wigner function

- Ansatz for global equilibrium distribution function at zero temperature:

$$\nu^0(\mathbf{r},\mathbf{k})=\Theta\left(1-\sum_i \frac{r_i^2}{R_i^2}-\sum_i \frac{k_i^2}{K_i^2}\right)$$

  R_i and K_i are Thomas-Fermi radii and momenta

- Scaling ansatz:

$$\nu(\mathbf{r},\mathbf{k},t)\to\Gamma(t)\nu^0(\boldsymbol{\mathcal{R}}(\mathbf{r},t),\boldsymbol{\mathcal{K}}(\mathbf{r},\mathbf{k},t))$$

Rescaled variables: $\mathcal{R}_i(\mathbf{r},t)=\frac{r_i}{b_i(t)}$ and $\mathcal{K}_i(\mathbf{r},\mathbf{k},t)=\frac{1}{\sqrt{\theta_i(t)}}\left[k_i-\frac{M\dot{b}_i(t)r_i}{\hbar b_i(t)}\right]$

- Normalization factor:

$$\Gamma(t)^{-1}=\prod_i b_i(t)\sqrt{\theta_i(t)}$$

Phys. Rev. A **68**, 043608 (2003)

Equations

- Equations of motion for scaling parameters:

$$\ddot{b}_i+\omega_i^2 b_i-\frac{\hbar^2 K_i^2\theta_i}{M^2 b_i R_i^2}+\frac{48Nc_0}{Mb_i R_i^2\prod_j b_j R_j}\left[f\left(\frac{b_x R_x}{b_z R_z},\frac{b_y R_y}{b_z R_z}\right)-b_i R_i\frac{\partial}{\partial b_i R_i}f\left(\frac{b_x R_x}{b_z R_z},\frac{b_y R_y}{b_z R_z}\right)\right]$$

$$-\frac{48Nc_0}{Mb_i R_i^2\prod_j b_j R_j}\left[f\left(\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_x}K_x},\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_y}K_y}\right)+\sqrt{\theta_i}K_i\frac{\partial}{\partial\sqrt{\theta_i}K_i}f\left(\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_x}K_x},\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_y}K_y}\right)\right]=0,$$

$$\dot{\theta}_i+2\frac{\dot{b}_i}{b_i}\theta_i=-\frac{1}{\tau}(\theta_i-\theta_i^{\mathrm{hy}})$$

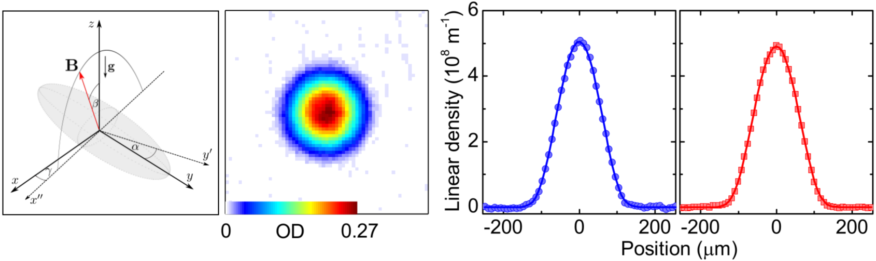
- Strength of dipolar interaction:

$$c_0=\frac{2^{10}C_{\mathrm{dd}}}{3^4\cdot 5\cdot 7\cdot\pi^3}$$

- Collisionless regime: $\tau\to\infty$
- Hydrodynamic regime: $\tau\to 0$, definition: $\prod_i b_i^{\mathrm{hy}}(t)\sqrt{\theta_i^{\mathrm{hy}}(t)}=1$

Phys. Rev. A **96**, 043608 (2017)

Innsbruck experiment



Science **345**, 1484 (2014)

- Aspect ratios of the atomic cloud and FS in imaging plane:

$$A_R(t)=\sqrt{\frac{\langle r_{\mathrm{v}}^2(t)\rangle}{\langle r_{\mathrm{h}}^2(t)\rangle}}=\sqrt{\frac{\langle r_z^2(t)\rangle}{\langle r_x^2(t)\rangle\cos^2\alpha+\langle r_y^2(t)\rangle\sin^2\alpha}},\qquad \langle r_i^2(t)\rangle=\frac{1}{8}R_i^2 b_i^2(t)$$

$$A_K(t)=\sqrt{\frac{\langle k_{\mathrm{v}}^2(t)\rangle}{\langle k_{\mathrm{h}}^2(t)\rangle}}=\sqrt{\frac{\langle k_z^2(t)\rangle}{\langle k_x^2(t)\rangle\cos^2\alpha+\langle k_y^2(t)\rangle\sin^2\alpha}},\qquad \langle k_i^2(t)\rangle=\frac{1}{8}K_i^2\theta_i(t)+\frac{1}{\hbar^2}M^2R_i^2\dot{b}_i^2(t)$$

Veljić et al., New J. Phys. **20**, 093016 (2018)

Time-of-flight expansion

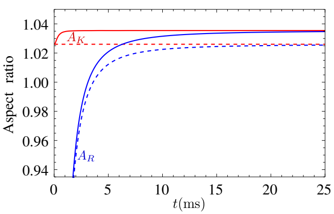
- Ballistic expansion:

$$A_K(0) = \lim_{t\to\infty} A_R(t)$$

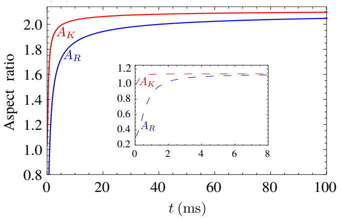
- Non-ballistic expansion:

$$\lim_{t\to\infty} A_K(t) = \lim_{t\to\infty} A_R(t)$$

Collisionless regime



Hydrodynamic regime

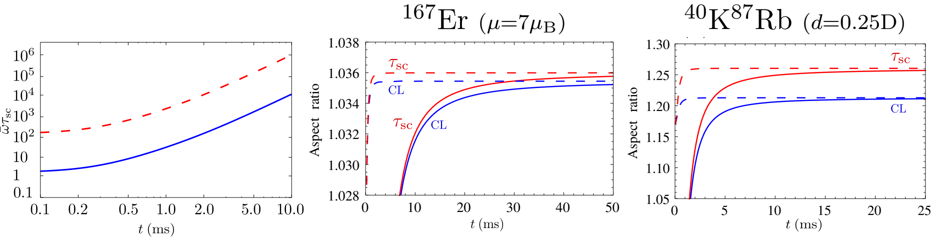


Veljić et al., Phys. Rev. A **95**, 053635 (2017)

Self-consistent τ

- Self-consistent relaxation time: $\tau_{\rm sc}(t) \sim \frac{1}{\overline{n}(t)}$
- Mean number density: $\overline{n}(t) = \frac{N}{V(t)}$
- Volume of the gas during the TOF: $V(t) = \frac{4\pi}{3}\prod_i R_i b_i(t)$

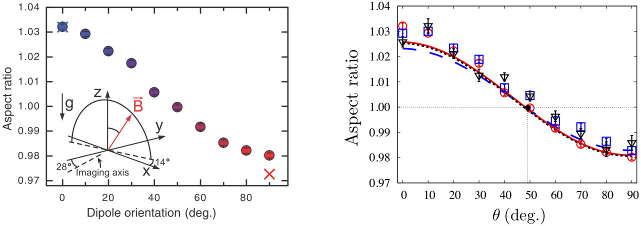
Phys. Rev. Lett. **113**, 263201 (2014); Phys. Rev. A **89**, 022702 (2014)



Veljić et al., Phys. Rev. A **95**, 053635 (2017)

Theory vs Experiment

- Comparison of theoretical results for A_{K} with the experimental results for $A_{\mathrm{R}}(t = 12\,\mathrm{ms})$ for ^{167}Er

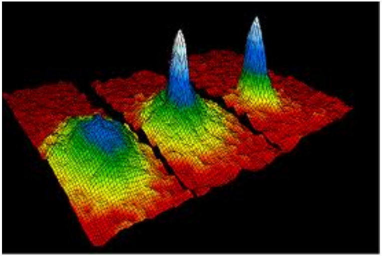


- $\varphi = 14^\circ$, $\alpha = 28^\circ$
- $N = 6.6 \cdot 10^4$, (579, 91, 611) Hz
- $N = 6.3 \cdot 10^4$, (428, 91, 459) Hz
- $N = 6.1 \cdot 10^4$, (408, 212, 349) Hz

Veljić et al., New J. Phys. **20**, 093016 (2018)

Bose-Einstein condensation

- Intensive progress in the field of ultracold atoms has been recognized by 2001 Nobel prize for experimental realization of Bose-Einstein condensation in 1995
- Cold alkali atoms:
  Rb, Na, Li, K ...
  $T \sim 1\,\mathrm{nK}$, $\rho \sim 10^{14}\mathrm{cm}^{-3}$
- Cold bosons, cold fermions
- Optical lattices
- Short-range interactions,
  long-range dipolar interactions



- Tunable quantum systems concerning dimensionality, type and strength of interactions

Feshbach resonance

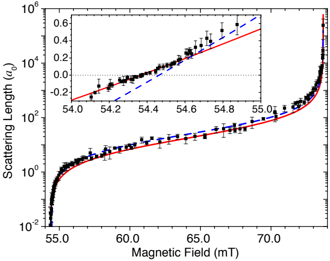
- Scattering length depends on the external magnetic field

- For ^{7}Li: PRL **102**, 090402 (2009)

$$a(B) = a_{\mathrm{BG}} \left(1 + \frac{\Delta}{B - B_\infty} \right)$$

$a_{\mathrm{BG}} = -24.5\, a_0$, $B_\infty = 73.68\,\mathrm{mT}$,
$\Delta = 19.2\,\mathrm{mT}$

- The interaction can be in principle tuned to any small or large, positive or negative value

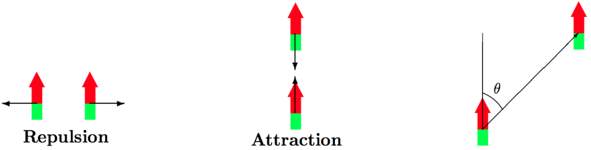


Dipolar Bose-Einstein condensates

- Interaction potential: contact plus dipole-dipole interaction

$$V_{\rm int}(\mathbf{r}) = g\delta(\mathbf{r}) + V_{\rm dd}(\mathbf{r})$$

$$g = \frac{4\pi\hbar^2 a_s}{m}, \qquad V_{\rm dd}(\mathbf{r}) = \frac{C_{\rm dd}}{4\pi|\mathbf{r}|^3}\left(1 - 3\cos^2\theta\right)$$



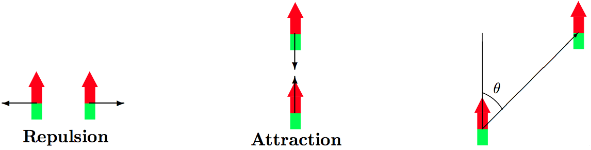
- Dipolar GPE at the MF level

Dipolar Bose-Einstein condensates

- Interaction potential: contact plus dipole-dipole interaction

$$V_{\text{int}}(\mathbf{r}) = g\delta(\mathbf{r}) + V_{\text{dd}}(\mathbf{r})$$

$$g = \frac{4\pi\hbar^2 a_s}{m}, \qquad V_{\text{dd}}(\mathbf{r}) = \frac{C_{\text{dd}}}{4\pi|\mathbf{r}|^3}\left(1 - 3\cos^2\theta\right)$$



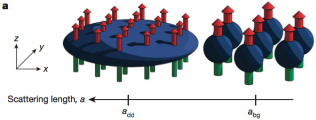
- Dipolar GPE at the MF level: nonlinear terms due to the contact and the DDI

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{trap}}(\mathbf{r}) + gN|\Psi(\mathbf{r},t)|^2 + N\int d^3r'\, V_{\text{dd}}(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}',t)|^2\right]\Psi(\mathbf{r},t)$$

Beyond MF: quantum droplets

- Surprisingly, such a collapse can be avoided
- Experiment of Pfau group in 2016 observed spontaneous transition from an unstructured superfluid to an ordered arrangement of droplets in ^{164}Dy BEC

Kadau et al., Nature **530**, 194 (2016)



- Local stability analysis shows that three-body interactions are not sufficient to describe droplet formation

Ferrier-Barbut et al., PRL **116**, 215301 (2016)

- Quantum fluctuations remain as main effect

L. Chomaz et al., PRX **6**, 041039 (2016)

Quantum droplets, supersolidity

- Dipolar droplets are predicted and shown to be self-bound:

  Baillie et al., PRA **94**, 021602 (2016); Schmitt et al., Nature **539**, 259 (2016)

- Droplet structures or macrodroplets:

  Chomaz et al., PRX **6**, 041039 (2016)

- Roton mode observation:

  Chomaz et al., Nat. Phys. **14**, 442 (2018); Schmidt et al., PRL **126**, 193002 (2021)

- Supersolidity in droplet arrays:

  Böttcher et al., PRX **9**, 011051 (2019); Chomaz et al., PRX **9**, 021012 (2019)

- Useful reviews:

  Böttcher et al., Rep. Prog. Phys. **84**, 012403 (2021);
  Chomaz et al., Rep. Prog. Phys. **86**, 026401 (2023)

- Vortices in dipolar droplets: Klaus et al., Nat. Phys. **18**, 1453 (2022)

Quantum fluctuations: Bogoliubov theory

- Shift of the chemical potential:

$$\Delta\mu = \frac{32g}{3\sqrt{\pi}}\,(na_s)^{3/2}\,\mathcal{Q}_5(\epsilon_{\rm dd})\,, \quad \mathcal{Q}_\ell(x) = \int_0^1 du\,\{1-x+3xu^2\}^{\ell/2}$$

PRA **84**, 041604(R) (2011); PRA **86**, 063609 (2012); PR **106**, 1135 (1957)

- Effective Gross-Pitaevskii equation within LDA

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm trap}(\mathbf{r}) + gN|\Psi|^2 + N\int d^3r'\,V_{\rm dd}(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}',t)|^2 + V_{\rm eff}\right]\Psi$$

$$V_{\rm eff} = \frac{32ga_s^{3/2}}{3\sqrt{\pi}}\,\mathcal{Q}_5(\epsilon_{\rm dd})\,|\Psi|^3$$

- This does not take into account condensate depletion due to excitations of atoms

Quantum fluctuations: Bogoliubov-Popov theory

- Condensate depletion:

$$n = n_0 + \Delta n\,, \quad n_0 = |\Psi|^2\,, \quad \Delta n = \frac{8}{3\sqrt{\pi}}\,(n_0 a_s)^{3/2}\,\mathcal{Q}_3(\epsilon_{\rm dd})$$

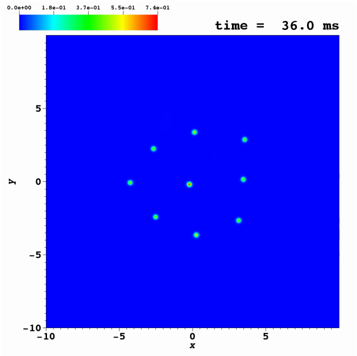
- Shift of the chemical potential:

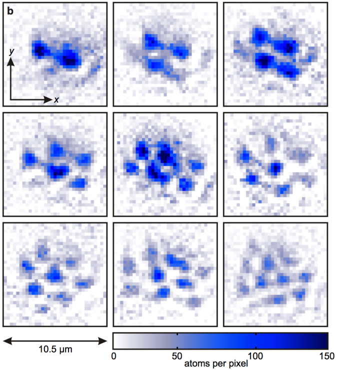
$$\Delta\mu = \frac{8g}{3\sqrt{\pi}}\,(n_0 a_s)^{3/2}\left\{4\mathcal{Q}_5(\epsilon_{\rm dd}) + \mathcal{Q}_3(\epsilon_{\rm dd})\,\frac{\widetilde{V}_{\rm int}(\mathbf{q}=0)}{g}\right\}$$

- Effective GPE within LDA:

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm trap}(\mathbf{r}) + g{\color{red}n} + N\int d^3r'\,V_{\rm dd}(\mathbf{r}-\mathbf{r}'){\color{red}n(\mathbf{r}',t)} + V_{\rm eff}\right]\Psi$$

Droplet formation: contact interaction quench





$$N = 20 \times 10^3,\ a_s = 131a_0 \to 62a_0,\ {}^{164}\mathrm{Dy},\ a_{\mathrm{dd}} = 131a_0,$$
$$2\pi \times (44,\ 46,\ 133)\ \mathrm{Hz}$$

Comput. Phys. Commun. **195**, 117 (2015); Comput. Phys. Commun. **200**, 406 (2016); Comput. Phys. Commun. **209**, 190 (2016); Comput. Phys. Commun. **286**, 108669 (2023)

Gradient corrections

- If we take into account inhomogeneity of the system due to existence of a trap and localized droplets, the Hamiltonian gets gradient corrections:

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + gn + \int d^3r'\, V_{\rm dd}(\mathbf{r}-\mathbf{r}')n(\mathbf{r}',t) + V_{\rm eff}\right]\Psi + \frac{\hbar^2 a_s^{3/2}}{m}\sum_{i=1}^{5} G_{\rm eff}^{(i)}$$

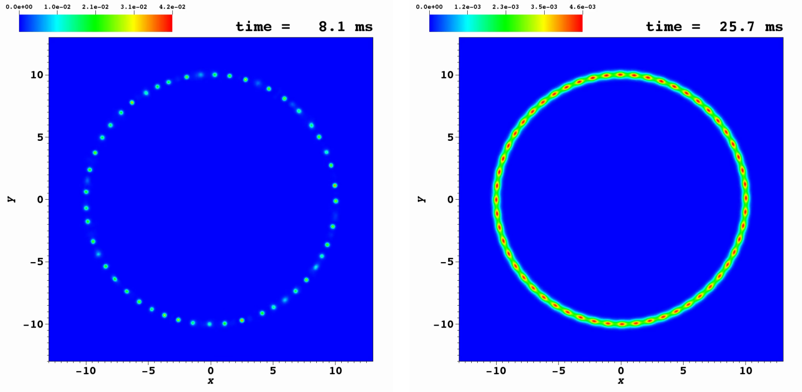
where $\Psi = |\Psi|\, e^{i\varphi}$, and

$$G_{\rm eff}^{(1)} = e^{-i\varphi}\sum_{k=1}^{3} C_k^{(1)}(\epsilon_{\rm dd})(\partial_k\Psi)^2\,,\; G_{\rm eff}^{(2)} = e^{3i\varphi}\sum_{k=1}^{3} C_k^{(2)}(\epsilon_{\rm dd})(\partial_k\Psi^*)^2$$

$$G_{\rm eff}^{(3)} = e^{i\varphi}\sum_{k=1}^{3} C_k^{(3)}(\epsilon_{\rm dd})(\partial_k\Psi)\partial_k\Psi^*\,,\; G_{\rm eff}^{(4)} = |\Psi|\sum_{k=1}^{3} C_k^{(4)}(\epsilon_{\rm dd})\partial_k^2\Psi$$

$$G_{\rm eff}^{(5)} = |\Psi|\, e^{2i\varphi}\sum_{k=1}^{3} C_k^{(5)}(\epsilon_{\rm dd})\partial_k^2\Psi^*$$

Novel phase of matter: supersolid



$N = 30,000$ $\quad$ $a_s = 132a_0 \to 62a_0$

$N = 30,000$ $\quad$ $a_s = 132a_0 \to 72a_0$

Conclusions

Thank you all and

happy birthday to Maja!