GRAVITATION AND GAUGE SYMMETRIES

Milutin Blagojević†

Institute of Physics
P.O.Box 57, 11001 Belgrade, Yugoslavia

April, 2001

† Email: mb@phy.bg.ac.yu

CONTENTS

PREFACE

The idea of a unified description of the basic physical interactions has evolved parallel to the development of our understanding of their dynamical structure. It has origins in Maxwell's unification of electricity and magnetism in the second half of the last century, matured in Weyl's and Kaluza's attempts to unify gravity and electromagnetism at the beginning of this century, and achieved the full development in the 70's, in the process of unification of the weak and electromagnetic, and also, to some extent, strong interactions. The biggest barrier to this attractive idea comes from a continual resistance of gravity to join the other basic interactions in the framework of a unified, consistent quantum theory.

In the course of the development of electromagnetic, weak and strong interactions, the concept of (internal) gauge invariance grew up and established itself as an unavoidable dynamical principle in particle physics. It is less known that the principle of equivalence, one of the prominent characteristics of the gravitational interaction, can be also expressed as a (spacetime) gauge symmetry. This book is intended to shed light upon the connection between the intrinsic structure of *gravity* and the principle of *gauge invariance*, which may lead to a consistent *unified* theory.

The first part of this book, chapters I–VI, gives a systematic account of the structure of gravity as a theory based on spacetime gauge symmetries. Some basic properties of space, time and gravity are reviewed in the first, introductory chapter. Chapter II deals with elements of the global Poincaré and conformal symmetries, which are necessary for the exposition of their localization; the structure of the corresponding gauge theories is explored in chapters III and IV. Then, in chapters V and VI, we present basic features of the Hamiltonian dynamics of Poincaré gauge theory, discuss the relation between gauge symmetries and conservation laws, and introduce the concept of the gravitational energy, and other conserved charges. The second part of the book treats the most promising attempts to build a unified field theory containing gravity, on the basis of the gauge principle. In chapters VII and VIII we discuss the possibility of constructing gravity as a field theory in flat spacetime. Chapters IX, X and XI yield an exposition of the ideas of supersymmetry and supergravity, Kaluza–Klein theory and string theory — these ideas can hardly be avoided in any attempt to build a unified theory of basic physical interactions.

This book is intended to provide a pedagogical survey of the subject of gravity from the point of view of particle physics and gauge theories, at the graduate level. The book is written as a self–contained treatise, which means that I assume no prior knowledge of gravity and gauge theories on the part of the reader. Of course, some familiarity with these subjects will certainly facilitate the reader to follow the exposition. Although the gauge approach differs from the more standard geometric approach, it leads to the same mathematical and physical structures.

The first part of the book (chapters II–VI) has evolved from the material covered in the one semester graduate course *Gravitation II*, taught for about twenty years at the University of Belgrade. Chapters IX–XI have been used as the basis for a one semester graduate course on unification of fundamental interactions.

Special features. The following remarks are intended to help the reader in an efficient use of the book.

- Examples in the text are used to illustrate and clarify the main exposition.
- The exercises given at the end of each chapter are an integral part of the book. They are aimed at illustrating, completing, applying and extending the results discussed in the text.
- Short comments on some specific topics are given at the end of each chapter, in order to illustrate the relevant research problems and methods of investigation.
- The Appendix consists of thirteen separate sections (A–M), which are in different relations to the main text.
  - Technical appendices J and M (Dirac spinors, Fourier expansion) are *indispensable* for the exposition in chapters IX and XI.
  - Appendices A, H and I (internal local symmetries, Lorentz and Poincaré group) are *very useful* for the exposition in chapters III (A) and IX (H, I).
  - Appendices C, D, E, F, G and L (de Sitter gauge theory, the scalar–tensor theory, Ashtekar's formulation of GR, constraint algebra and gauge symmetries, covariance, spin and interaction of massless particles, and Chern–Simons gravity) are *supplements* to the main exposition, and may be studied according to the reader's choice.
  - The material in appendices B and K (differentiable manifolds, symmetry groups and manifolds) is *not necessary* for the main exposition. It gives a deeper mathematical foundation for the geometric considerations in chapters III, IV and X.
- The bibliography contains references that document the material covered in the text. Several references for each chapter, which I consider as the most suitable for further reading, are denoted by the symbol •.
- Chapters IV, VII and VIII can be omitted in the first reading, without influencing the internal coherence of the exposition. Chapters IX, X

and XI are largely independent of each other, and can be read in any order.

Acknowledgments. The material presented in this book has originated under a strong influence of the research activities in the Belgrade group for *Particles and Fields*, during the last twenty years. I would like to mention here some of my teachers and colleagues which, in one or another way, significantly influenced my own understanding of gravity. They are: Rastko Stojanović and Marko Leko, my first teachers of tensor calculus and general relativity; Djordje Živanović, an inspiring mind for many of us who were fascinated by the hidden secrets of gravity; Paul Senjanović, who brought the Dirac canonical approach all the way from City College in New York to Belgrade; Ignjat Nikolić and Milovan Vasilić, first my students and afterwards respected collaborators; Branislav Sazdović, who silently introduced supersymmetry and superstrings in our everyday life; then, Dragan Popović, the collaborator from an early stage of our studies of gravity, and Djordje Šijački, who never stopped insisting on the importance of symmetries in gravity.

For illuminating comments and criticisms on a preliminary version of the manuscript, I wish to thank my colleagues Tatjana Vukašinac, Milovan Vasilić, Djordje Šijački, Branislav Sazdović, Nenad Manojlović, Aleksandar Bogojević, Friedrich Hehl and Eckehard Mielke. I am especially grateful for their valuable help to my students Olivera Mišković and Dejan Stojković, who carefully studied the first version of the manuscript and solved all the exercises with patience. I owe special thanks to Ignjat Nikolić, coauthor of the manuscript *The theory of gravity II*, written in 1986 for the graduate school of physics in Belgrade, for allowing me to use a large part of my contribution to that text, and include some of his formulations (in sections V.2 and VIII.4) and exercises. Finally, it is my pleasure to thank Friedrich Hehl and Yuri Obukhov for their sincere support during this project.

The exposition of Poincaré gauge theory and its Hamiltonian structure is essentially based on the research done in collaboration with Ignjat Nikolić and Milovan Vasilić. The present book incorporates a part of the spirit of that collaboration, which has been born during our long lasting joint work on the problems of gravity.

The subjects of the teleparallel theory, Chern–Simons gauge theory and three–dimensional gravity are written as an update to the first version of the manuscript (published in 1997, in Serbian). I would like to thank Antun Balaž and Olivera Mišković for their assistance at this stage, and to express my gratitude for the hospitality at the Primorska Institute for Natural Sciences and Technology, Koper, Slovenia, where a part of the update was completed.

Belgrade, April 2001 Milutin Blagojević