Accelerating Black Holes in 2+1D

Gabriel Arenas-Henriquez
Department of Mathematical Sciences, Durham University

Based on: GAH, R. Gregory, A. Scoins [arXiv:2202.08823]
GAH, A. Cisterna, F. Diaz, R. Gregory [In Progress]

2023 Workshop on Gravity, Holography, Strings and Noncommutative Geometry
Belgrade, Serbia
Feb 2023

Accelerating Black Holes

- Accelerating black holes in 3+1 dimensions.
- C-metric in 2+1 dimensions
  - Class I solutions: Accelerating particles
  - Class II solutions: Accelerating BTZ
  - Class Ic: Accelerating Black Hole (not quite like BTZ)
- Holographic description
  - 3+1
  - 2+1
- Concluding remarks

Accelerating black holes in 3+1

- To accelerate a black hole, we need to be able to push or pull it.
- Anything touching the event horizon must fall in unless traveling at the speed of light.
- This means that the physical object that touches the horizon must have energy = tension ($T^0_0 \sim T^1_1$)
- Fortunately, there is a candidate ➡ Cosmic String!

Cosmic string

- Very thin quasilinear object, which is fully characterised by its mass per unit length μ.

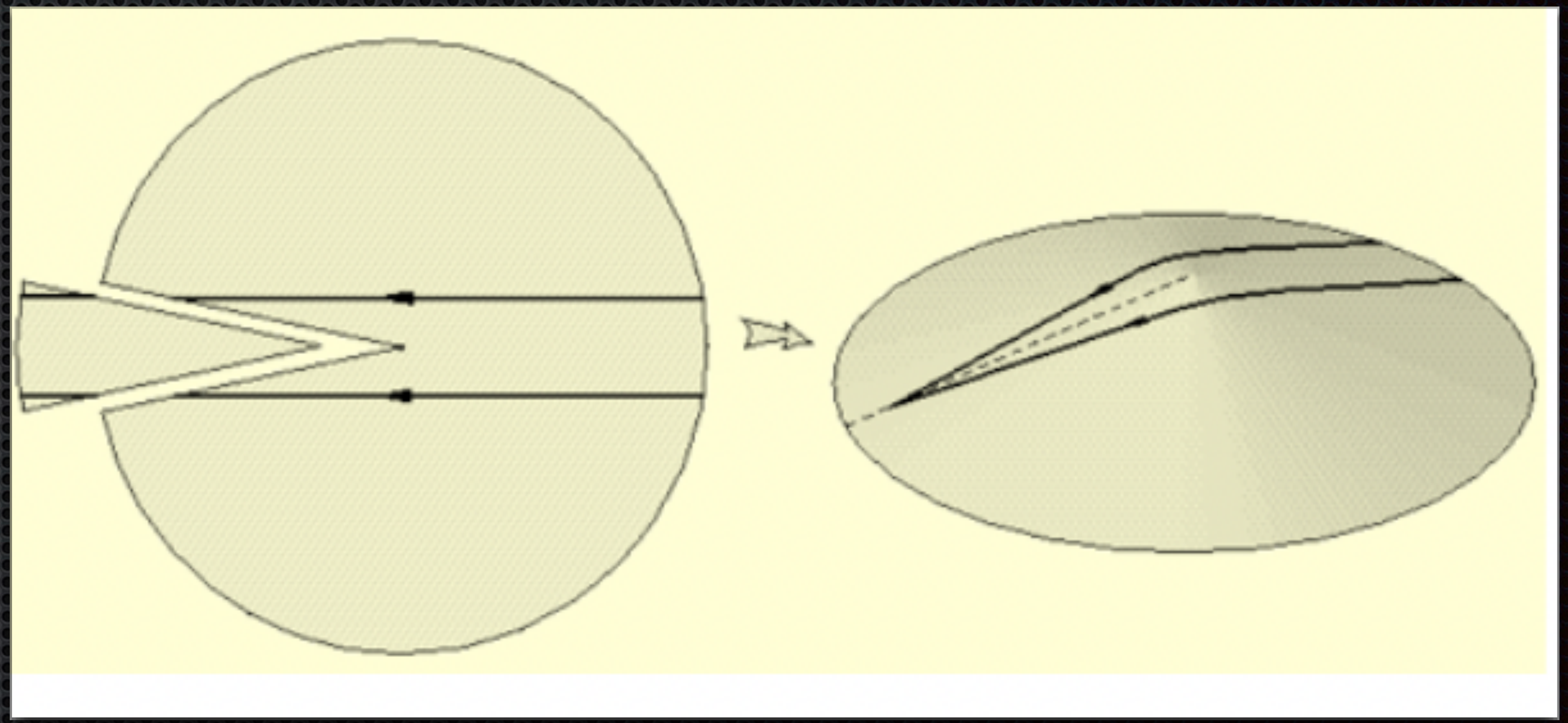
$$T^{\mu}_{\nu} \simeq \delta^{(2)}(r)\mathrm{diag}(\mu, \mu, 0, 0)$$

- The string produces a conical defect

$$\delta = 8\pi G\mu$$

The conical defect is what accelerates the black hole!

- No long range spacetime curvature.



The C-metric

- Let's consider the Plebanski-Demianski metric with non-zero cosmological constant $\Lambda = -\dfrac{3}{\ell^2}$ parametrised by two parameters

$$ds^2 = \frac{1}{A^2(x-y)^2}\left(-P(y)dt^2 + \frac{1}{P(y)}dy^2 + \frac{1}{Q(x)}dx^2 + Q(x)dz^2\right)$$

Griffiths and Podolsky 2006'

$$P(y) = \left(\frac{1}{A^2\ell^2} - 1\right) + y^2 - 2mAy^3\,,$$

$$Q(x) = 1 - x^2 - 2mAx^3$$

- The (conformal) boundary $x = y$
- Useful form to see range of the variables that preserves Lorenzian signature
- Not so clear view of black hole structure

- Introduce radial and angular variables $y = \dfrac{-1}{Ar}$, $x = \cos\theta$, $z = \dfrac{\phi}{K}$ and rescaling the time $t \to tA$, we get

$$ds^2 = \frac{1}{\Omega}\left[-f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

Hong and Teo 2003'

$$f(r) = \left(1 - \frac{2m}{r}\right)(1 - A^2r^2) + \frac{r^2}{\ell^2}, \quad g(\theta) = 1 + 2mA\cos\theta\,, \quad \Omega(r,\theta) = 1 + Ar\cos\theta$$

- The conformal factor blows up at $r = \dfrac{-1}{A\cos\theta}$
- The horizon(s) structure is determined by an interplay between m, A and ℓ
  - If A is big enough, there is an accelerating horizon (+ the black hole horizon)
  - If $A^2\ell^2 < 1$, we have a single black horizon ($r_h \sim 2m$)
- K determines the conical singularity $\delta = 2\pi\left(1 - \dfrac{1}{K}\right) = 8\pi\mu$

Aryal, Ford, Vilenkin 1986'
Achucarro, Gregory, Kuijken 1995'

- More precisely, to produce the acceleration, we need an imbalance between the north and south pole of the black hole

$$\mu_{\pm} = 2\pi\left(1 - \frac{g(0)}{K}\right) = 2\pi\left(1 - \frac{1 \pm 2mA}{K}\right) = 8\pi\mu_{\pm}$$

- We can regularise one of the axis, viz. $\delta_N = 0 \Rightarrow K = 1 + 2mA$

$$\delta_S = \frac{2\pi mA}{K} \Rightarrow \mu_S = \frac{mA}{K}$$

- How can we see the effect of the acceleration in the system?

- We can see the effect of A by considering $m = 0$: $f(r) = 1 - A^2r^2 + \dfrac{r^2}{\ell^2}$, $g(\theta) \to 1$

$$ds^2 = \frac{1}{\Omega}\left[-\left(1 - A^2r^2 + \frac{r^2}{\ell^2}\right)dt^2 + \frac{1}{\left(1 - A^2r^2 + \frac{r^2}{\ell^2}\right)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

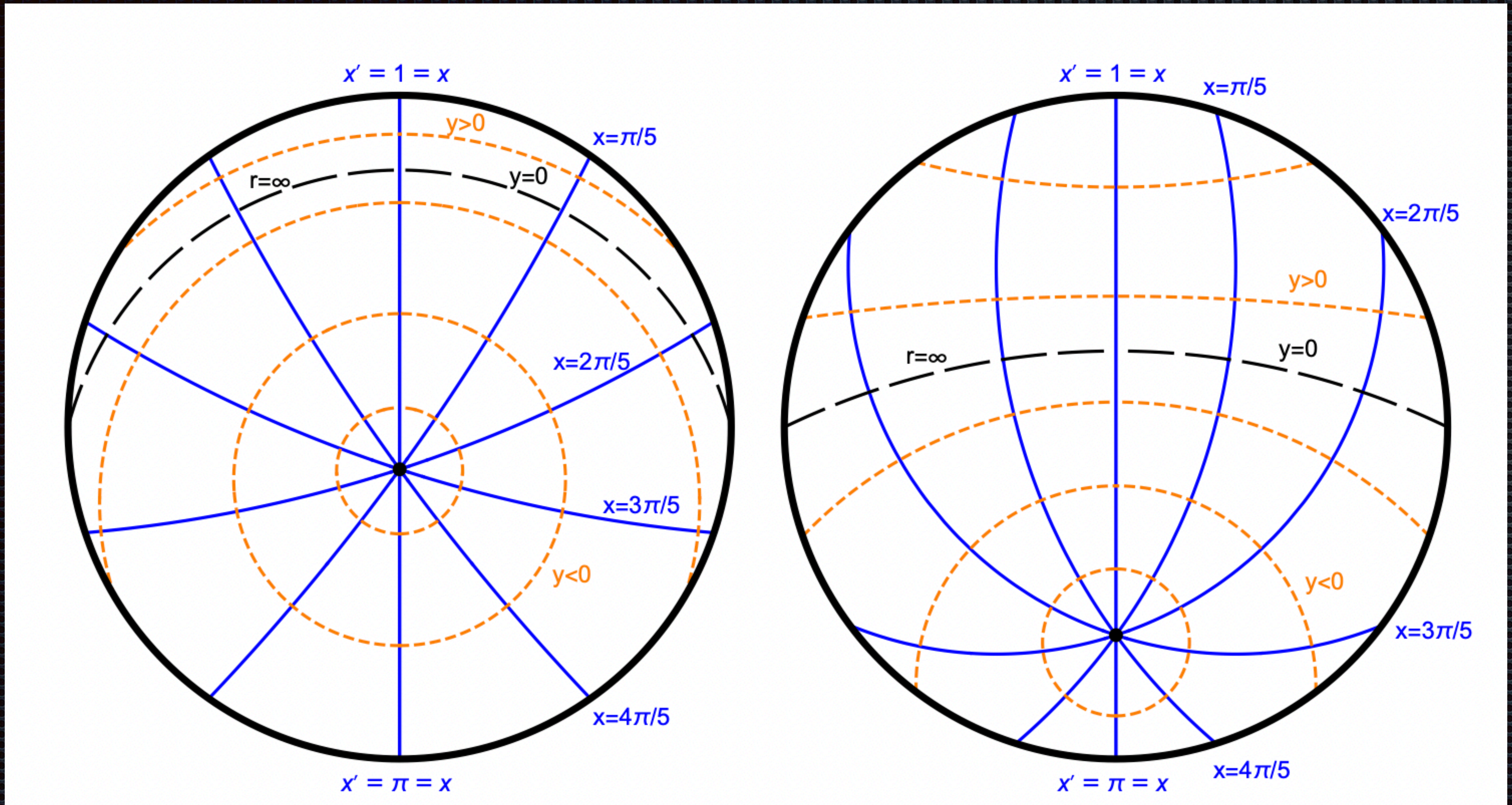
- In the slowly accelerating limit $A^2\ell^2 < 1$, we take introduce the following coordinates

$$1 + \frac{R^2}{\ell^2} = \frac{1 + (1 - A^2\ell^2)\frac{r^2}{\ell^2}}{\left(1 - A^2\ell^2\right)\Omega^2} \quad , \quad R\sin\Theta = \frac{r\sin\theta}{\Omega}$$

We get

$$ds^2 = -\left(1 + \frac{R^2}{\ell^2}\right)\beta dt^2 + \frac{1}{\left(1 + \frac{R^2}{\ell^2}\right)}dR^2 + R^2\left(d\Theta^2 + \sin^2\Theta\frac{d\phi^2}{K^2}\right)$$

$$\beta = \sqrt{1 - A^2\ell^2}$$



A. Scoins, 2022

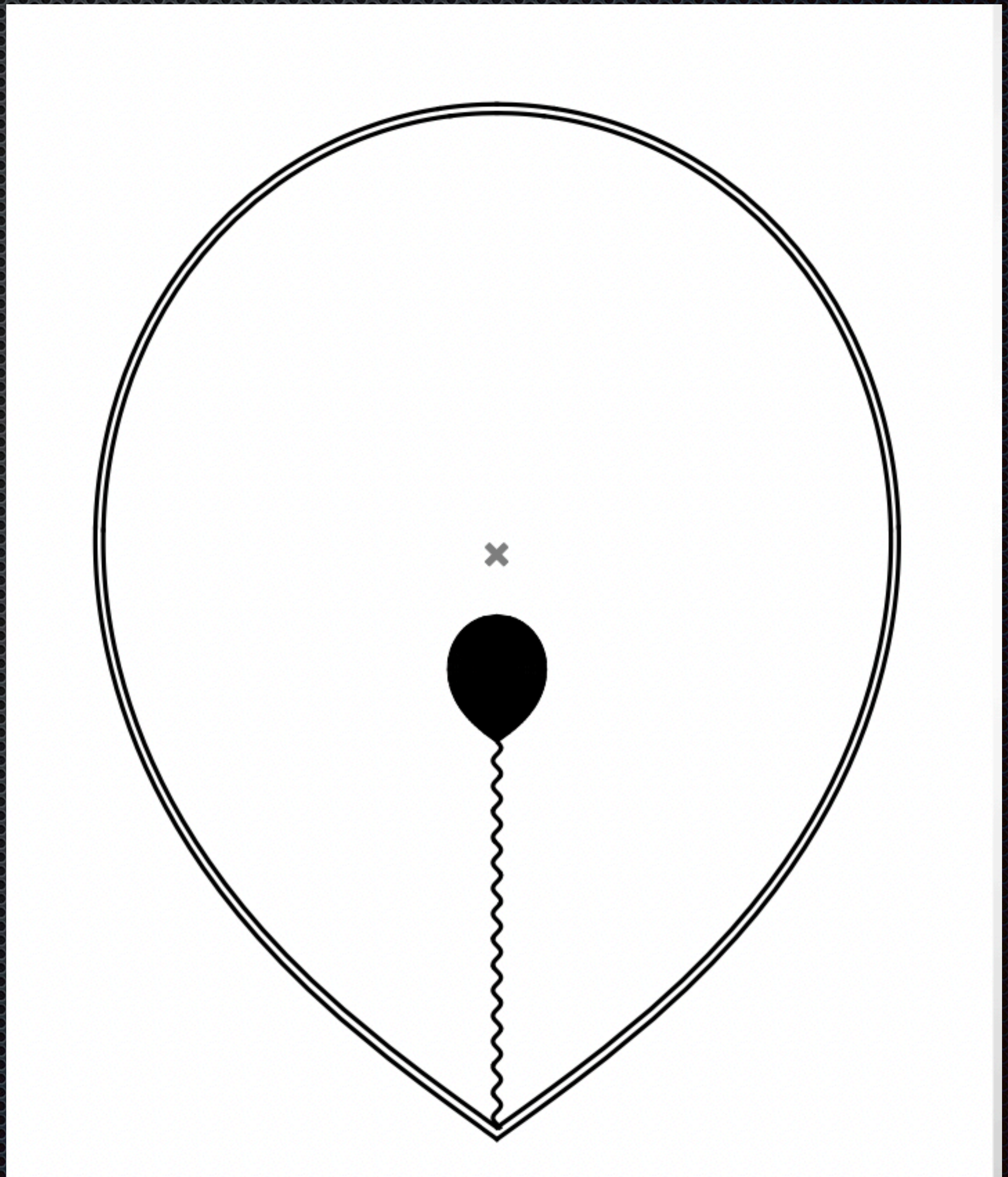
$$r = 0 \Leftrightarrow R = \frac{A^2\ell^4}{1 - A^2\ell^2}$$

$A\ell = 0.3$ $\qquad$ $A\ell = 0.9$

- We have an off-centre global AdS perspective. As we increase A, the point mass is "pulled" closer to the AdS boundary.

The slowly accelerating black hole in AdS is displaced from centre. It has a conical deficit running from the horizon to the boundary. The string tension provides the force that hold the black hole off-centre.

- The system is “accelerating” and yet, remains static suspended due to the string tension $\mu_s = mA/K \ (\leftrightarrow F = MA)$



Thermodynamics

- The temperature can be obtained through the standard Euclidean method

$$T = \frac{f'(r_+)}{4\pi\alpha} = \frac{1}{2\pi r_+^2 \alpha}\left[m(1 - A^2 r_+^2) + \frac{r_+^3}{\ell^2(1 - A^2 r_+^2)}\right]$$

- Entropy

$$S = \frac{\pi r_+^2}{K(1 - A^2 r_+^2)}$$

- What about the mass?

- Holographic mass: Fefferman-Graham expansion in a bit 'unusual' way

$$\frac{1}{r} = -A\xi - \sum_{m=1}^{\infty} F_m(\xi) z^m$$

$$\cos\theta = \xi + \sum_{m=1}^{\infty} G_m(\xi) z^m$$

- Where F_m and G_m are determined by requiring

$$ds^2 = \frac{\ell^2}{z^2} dz^2 + \frac{\ell^2}{z^2}\left(g_{(0)ij} + z^2 g_{(2)} + \cdots\right) dx^i dx^j$$

- We choose $F_1 = -\dfrac{(1-A^2X)^{3/2}}{A\omega(x)\alpha}$ with $X = (1-x^2)(1+2mAx)$, then

$$ds^2_{(0)} = -\omega^2 d\tau^2 + \frac{\omega^2\alpha^2\ell^2 dx^2}{X(1-A^2\ell^2X)^2} + \frac{X\omega^2\alpha^2\ell^2 d\phi^2}{K^2(1-A^2\ell^2X)}$$

- The Balasubramanian-Kraus stress tensor $\langle T_{ij}[g_{(0)}]\rangle = \lim_{z\to 0}\dfrac{1}{z^D-3}Tij[h]$,

$$\langle T^t_t\rangle = \rho_E = \frac{m}{8\pi\ell^2\alpha^3\omega^3}(1-A^2\ell^2X)^{3/2}(2-3A^2\ell^2X)$$

- So, the mass is $$M = \int dx d\phi \sqrt{-g_{(0)}}\rho_E = \frac{\alpha m}{K} \qquad \alpha = \sqrt{1-A^2\ell^2}$$

Anabalon, Appels, Gregory, Kubiznak, Mann, Ovgün 2018

- Euclidean action

$$I_E = \frac{\beta}{2\alpha K}\left(m - 2mA^2\ell^2 - \frac{r_+^3}{\ell^2(1 - A^2 r_+^2)^2}\right) = \beta F$$

Anabalon, Appels, Gregory, Kubiznak, Mann, Ovgün 2018

- No Hawking-Page phase transition.
- Extended thermodynamics Gregory, Scoins 2019

C-metric in 2+1 dimensions

Anber 2008
Astorino 2011
GAH, Gregory, Scoins 2022

- We start with an ansatz similar to Plebanski-Demianski metric.

$$ds^2 = \frac{1}{A^2(x-y)^2}\left(-P(y)dt^2 + \frac{1}{P(y)}dy^2 + \frac{1}{Q(x)}dx^2\right)$$

- We eliminate all the gauge freedom of the metric functions getting quadratic eqs for $P(y)$ and $Q(x)$.
- We restrict the coordinates for ranges where Lorenzian signature holds.
- We get 3 classes of solutions depending on the range of x

Class	$Q(x)$	$P(y)$	Maximal range of x
I	$1-x^2$	$\frac{1}{A^2\ell^2}+(y^2-1)$	$\|x\|<1$
II	x^2-1	$\frac{1}{A^2\ell^2}+(1-y^2)$	$x>1$ or $x<-1$
III	$1+x^2$	$\frac{1}{A^2\ell^2}-(1+y^2)$	$\mathbb{R}$

- Finally, we insert a (single) domain wall by cutting and gluing the spacetime at $x = \mathbf{const}$. Israel's condition give the tension of the wall $\mu = \pm \frac{A}{4\pi}\sqrt{Q(x)}$.

- This yields to

Class I $\rightarrow$ Accelerating particles: pulled by a wall or pushed by a strut

Class I $\rightarrow$ Type I_C: An accelerating black hole pushed by a strut

Class II $\longrightarrow$ Accelerating BTZ: pulled by a wall or pushed by a strut

Class III $\longrightarrow$ No single wall solutions. Braneworld-type of geometries allowed.

Class I: Accelerating particles

- Using $r = \dfrac{-1}{Ay}$, $t = \alpha\dfrac{\tau}{A}$, $x = \cos\left(\dfrac{\phi}{K}\right)$

$$ds^2 = \frac{1}{[1 \pm Ar\cos\left(\phi/K\right)]^2}\left[-f(r)\frac{dt^2}{\alpha^2} + \frac{dr^2}{f(r)} + r^2\frac{d\phi^2}{K^2}\right]$$

$$f(r) = 1 + (1 - A^2\ell^2)\frac{r^2}{\ell^2}$$

Slow acceleration $A\ell < 1$ No Horizon!

Rapid acceleration $A\ell < 1$ Acc. Horizon

Tension $\sigma = \pm\dfrac{A}{4\pi}\sin\left(\dfrac{\pi}{K}\right)$

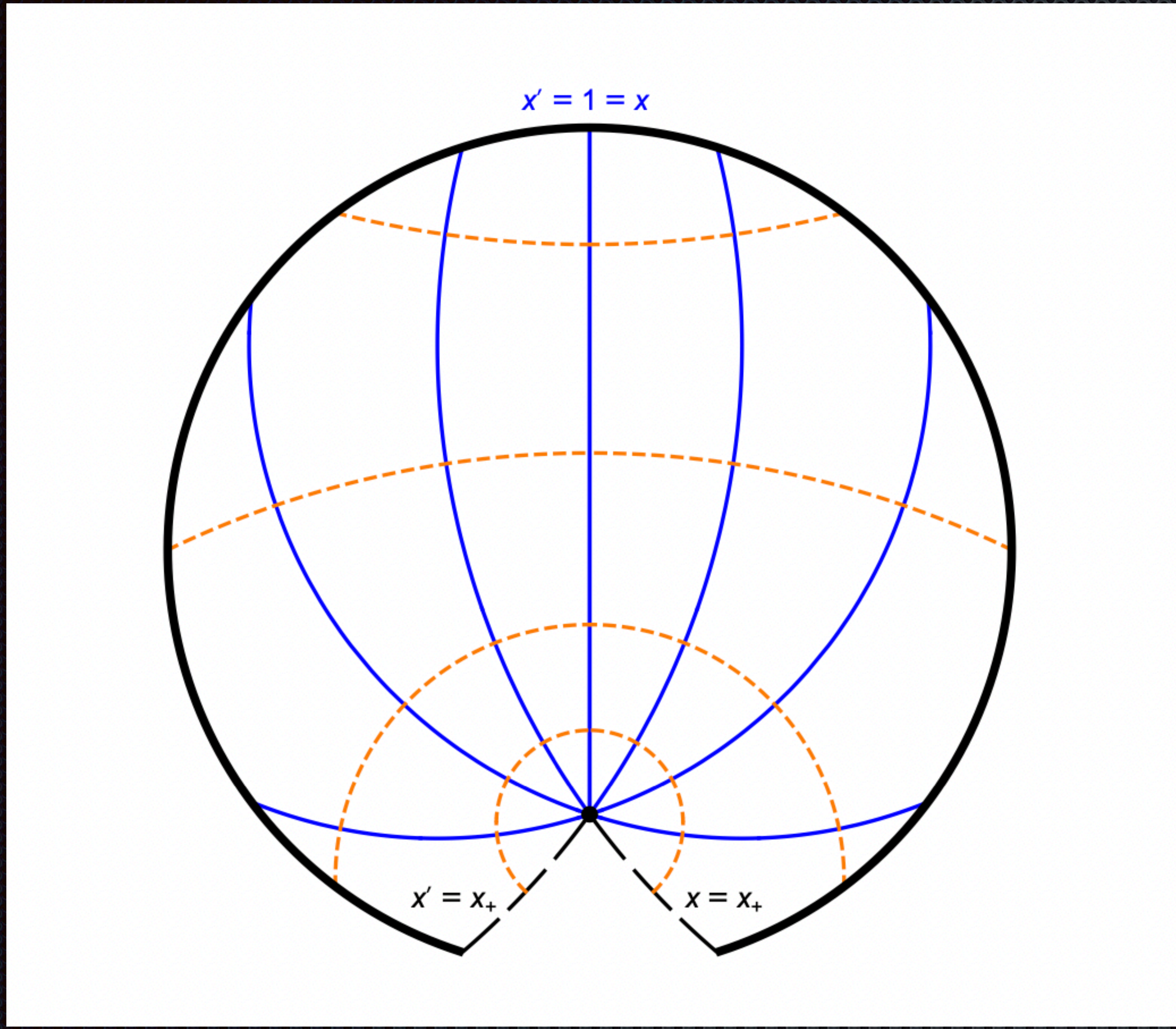
- Particle mass? We follow the same FG prescription as for 3+1d.

$$M = -\frac{1}{8\pi}\left(\frac{\pi}{2} - \arctan\left[\frac{\cot\left(\frac{\pi}{K}\right)}{\sqrt{1 - A^2\ell^2}}\right]\right)$$

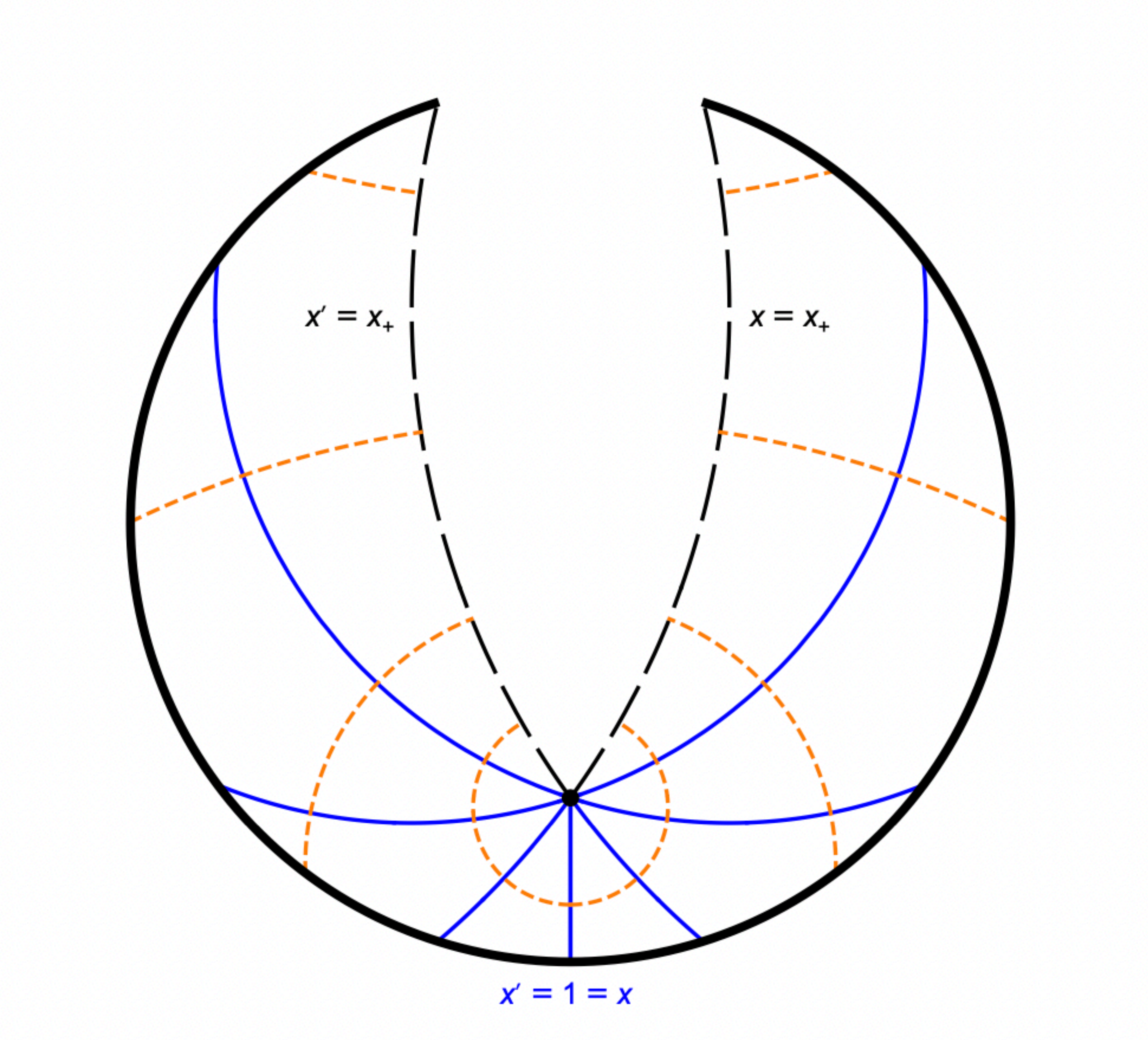
- For rapid acc. phase, there is temperature associated to the Rindler horizon $T = \dfrac{\sqrt{A^2\ell^2 - 1}}{2\pi\ell\alpha}$

$$S = \frac{\text{Area}}{4G} = -4\ell \operatorname{arctanh}\left[\left(-A\ell + \sqrt{A^2\ell^2 - 1}\right)\tan\left(\frac{\pi}{2K}\right)\right]$$

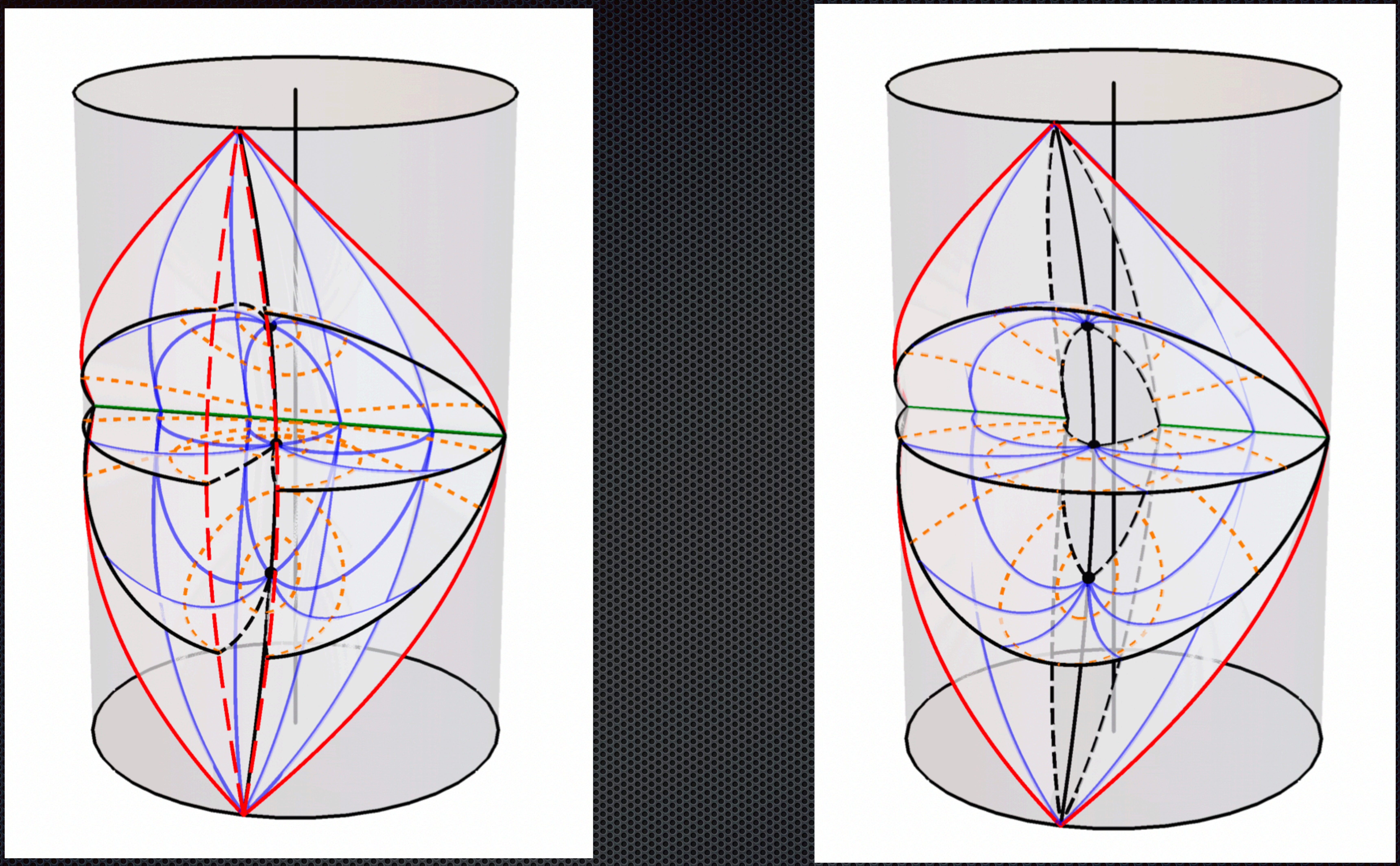
Class I: Accelerating particles $R_0 = \frac{A\ell^2}{\alpha}$



Slowly accelerating conical defect
Pulled by a wall $A = 0.9\ell$



Slowly accelerating conical defect
pushed by a strut $A = 0.9\ell$



Embedding within global AdS_3: The particle worldline is shown in solid black. Several surfaces of constant t are plotted. The event horizons are demonstrated by the surfaces at early and late t. The bifurcation surface is shown as a green line. The boundary of the classically accessible subset of the global boundary is shown in red. Lines of constant x are shown in blue, with lines of constant y in dashed orange.

Class II: Accelerating BTZ

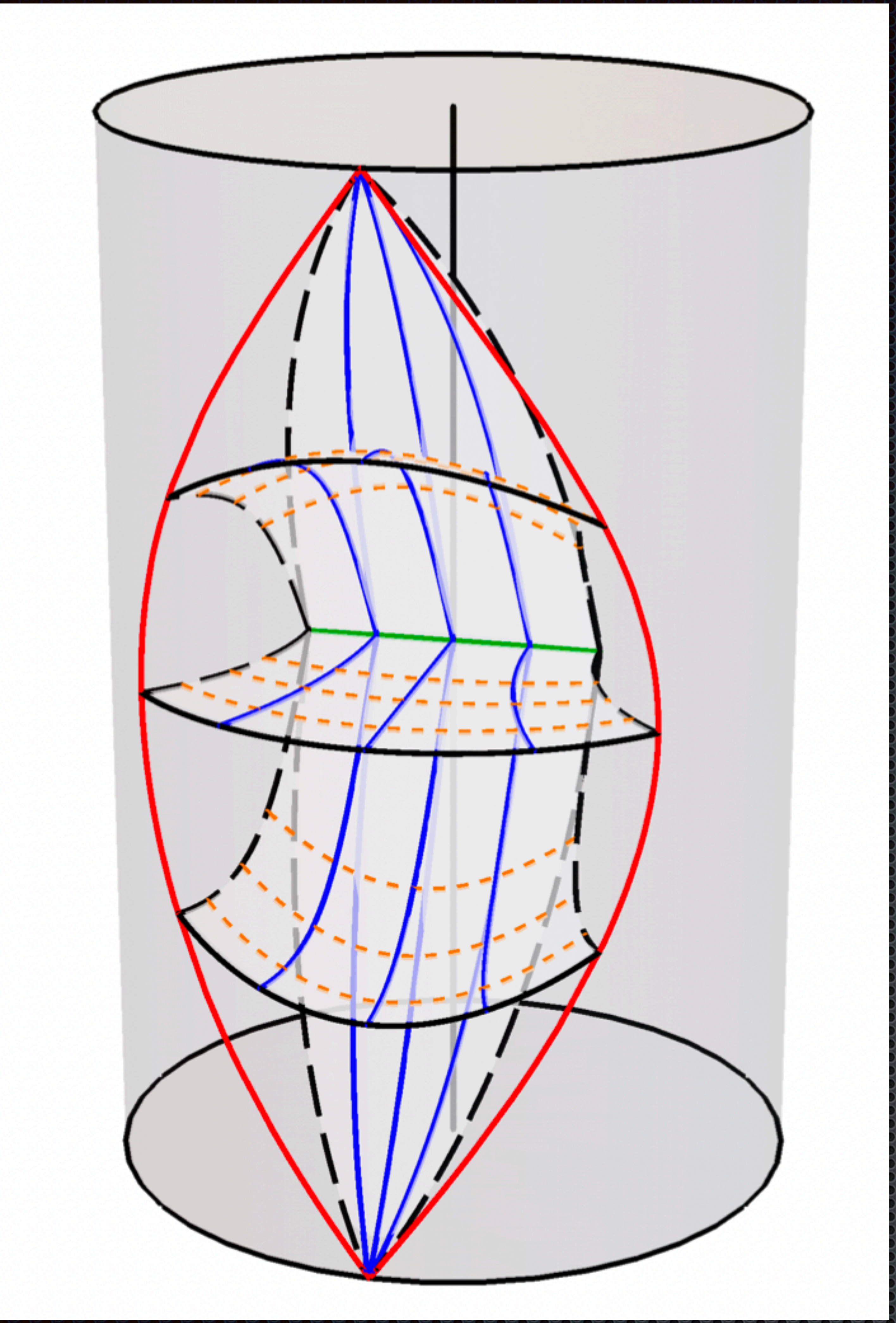
- We need to identify $x = \pm \cosh(\psi/K)$ where $\psi \in (-\pi, \pi)$, $r = -(Ay)^{-1}$, $t = \alpha A^{-1}\tau$

$$ds^2 = \frac{1}{\Omega}\left[-F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\psi^2\right]$$

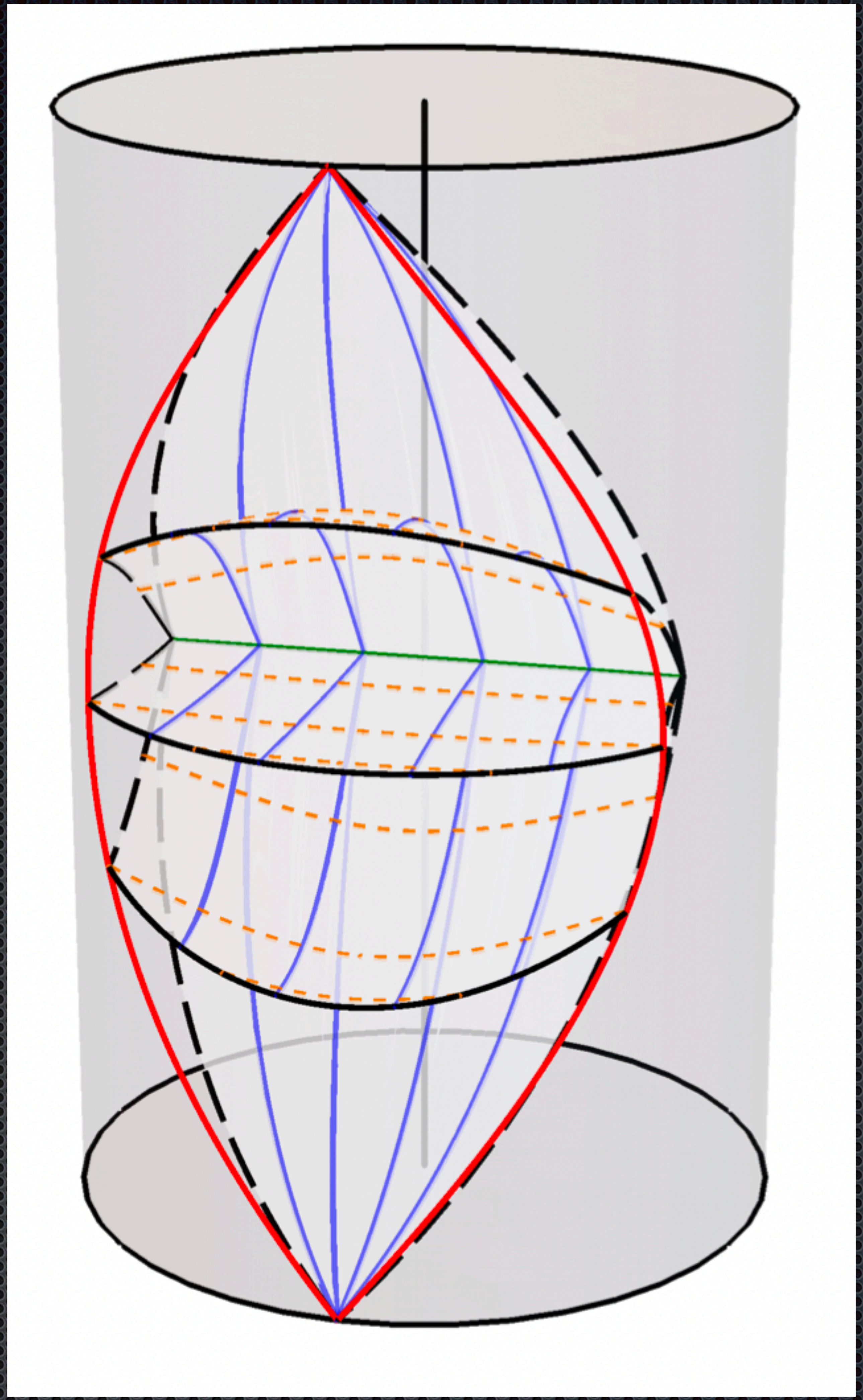
$$F(r) = -m^2\left(1 - A^2 r^2\right) + \frac{r^2}{\ell^2} \quad , \quad \Omega(r, \psi) = 1 \mp Ar\cosh\psi$$

- \+ Pulled by a wall
- \- Pushed by a strut

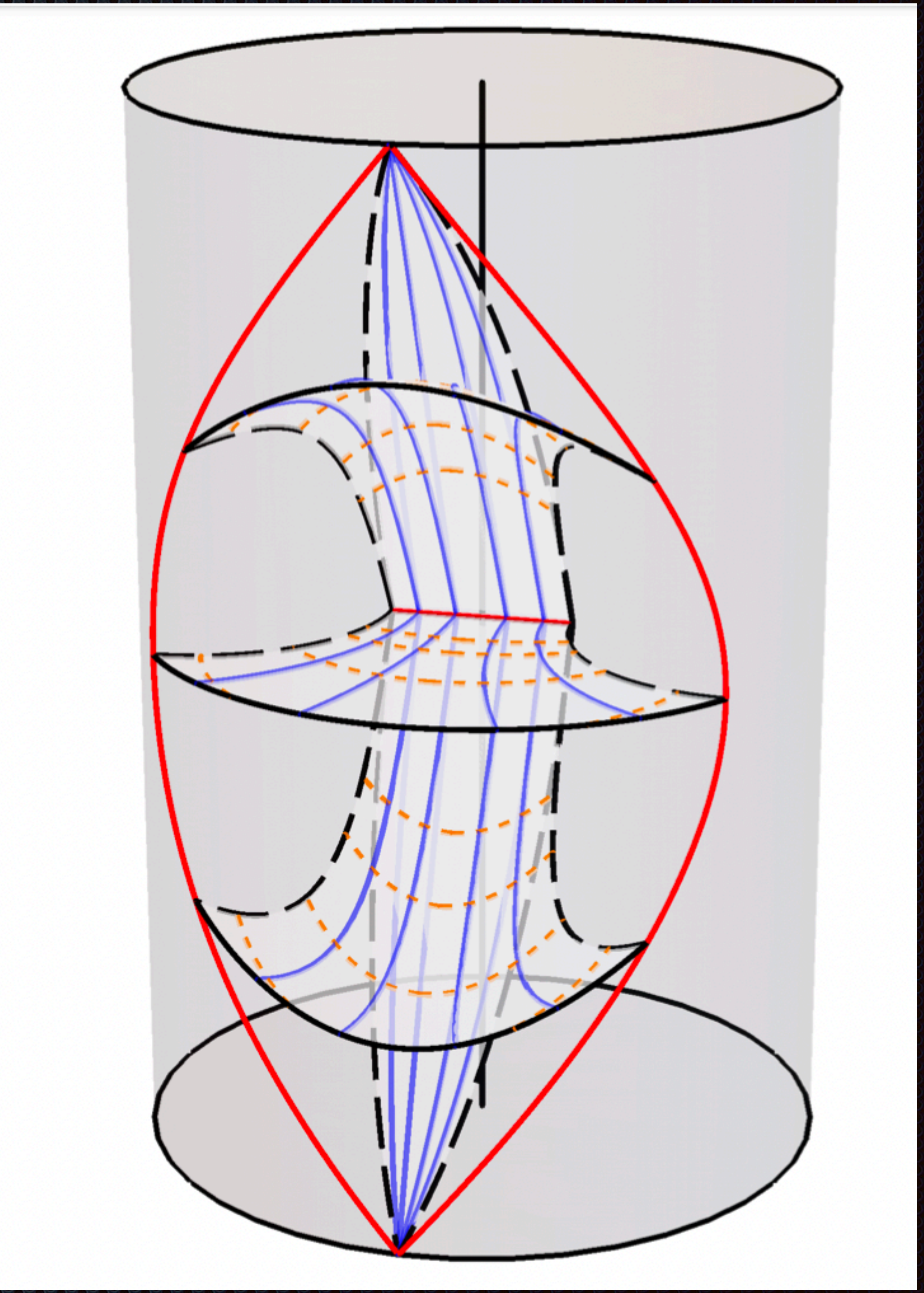
Where we also introduced the parameter $m = 1/K$, and $A \to mA$



Standard BTZ



BTZ pulled by a wall



BTZ pushed by a strut

Class I_C: A black hole pulled by a wall

When $A\ell > 1$, there is a horizon $y_h^2 = 1 - \dfrac{1}{A^2\ell^2}$

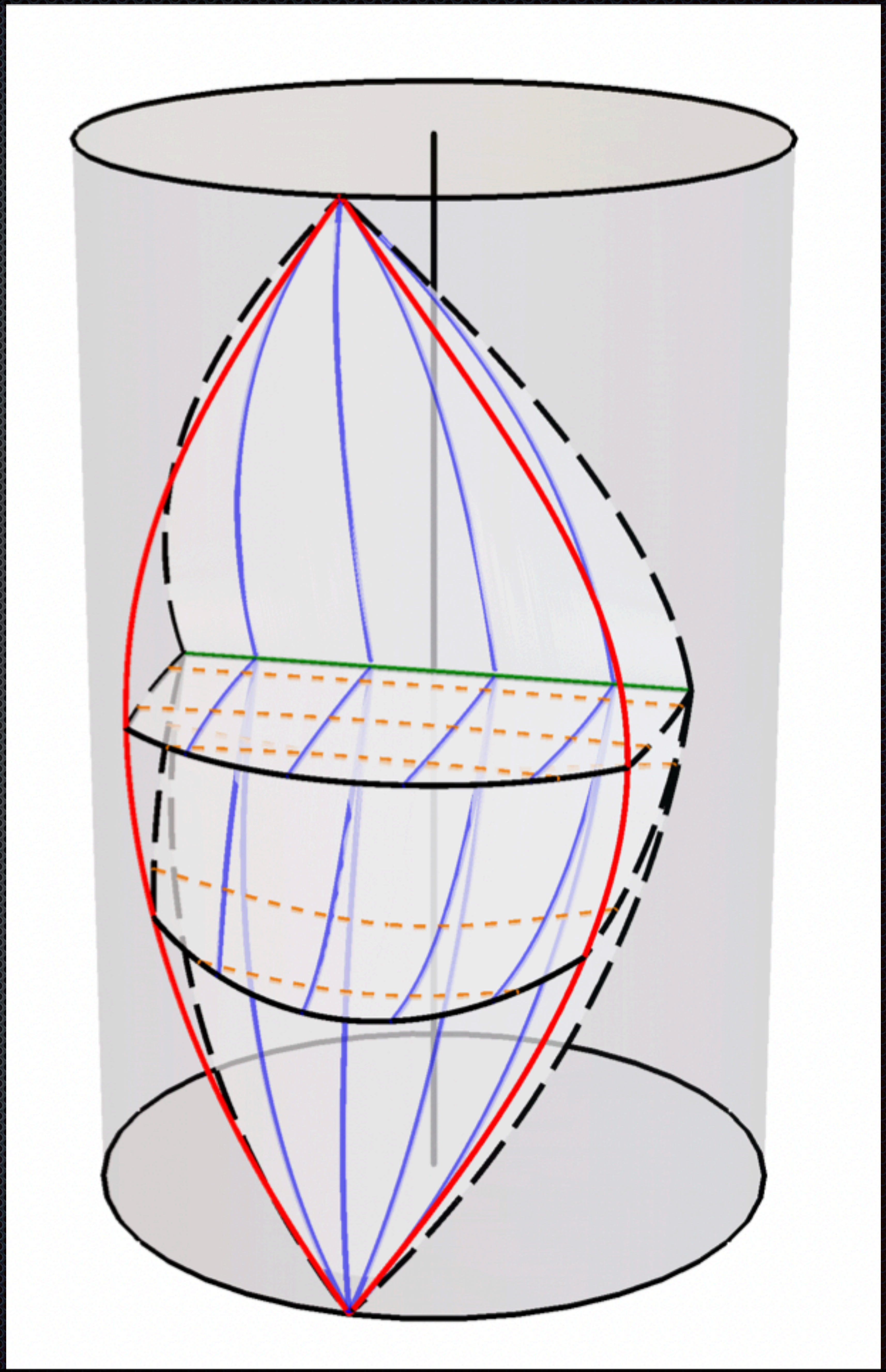
For the acc. Particle, we usually take $y < -y_h$, but we can also have $y \in (y_h, x)$, $x \in (x_+, 1)$

We identify $x = \pm\cos(\phi/K)$, $\rho = (Ay)^{-1}$, $t = \alpha A^{-1}\tau$

$$ds^2 = \frac{1}{\Omega}\left[-F(\rho)dt^2 + \frac{1}{F(\rho)}dr^2 + \rho^2 d\phi^2\right]$$

$$F(\rho) = -m^2\left(A^2\rho^2 - 1\right) + \frac{\rho^2}{\ell^2}, \quad \Omega(\rho,\phi) = Ar\cos m\phi - 1, \quad \frac{1}{m} \le A\ell < \frac{1}{m\sin(m\pi)}$$

In global coordinates shows a clear parallel with BTZ. However, there is no continuous link to the BTZ metric.



Holographic interpretation?

On going work with Cisterna, Diaz, Gregory

- In 3+1 dimensions, the holographic stress tensor

$$\langle T^i_j \rangle = \begin{pmatrix} -\rho_E & 0 & 0 \\ 0 & \Pi + \frac{\rho_E}{2} & 0 \\ 0 & 0 & \frac{\rho_E}{2} - \Pi \end{pmatrix}$$

Anabalon, Appels, Gregory, Kubiznak, Mann, Ovgün 2018

- With $\rho_E = \frac{m}{8\pi\ell^2\alpha^3\omega^3}(1 - A^2\ell^2 X)^{3/2}(2 - 3A^2\ell^2 X), \quad \Pi = \frac{3mA^2X}{16\pi\alpha^3\omega^3}(1 - A^2\ell^2 X)^{3/2}$

- Can be written in the fluid/gravity correspondence language

Rangamani 2009
Hubeny, Marolf, Rangamani 2010

- The fluid/gravity stress tensor $\langle T_{ab} \rangle = \Delta(x)\left(\frac{3}{2}u_a u_b + \frac{1}{2}g_{(0)ab}\right) + \Sigma_{ab}$

- With $\Sigma_{ab} = \xi\Theta_{ab} = \xi(C_{abd}u^d + C_{bad}u^d)$

Mukhopadhyay, Petkou, Petropoulos, Pozzoli, Siampos 14'

$\downarrow$

Cotton tensor $C_{abc} = \nabla_{[c}R_{b]a} - \frac{1}{4}g_{a[b}\nabla_{c]}R$

- Here $\Delta(x) = \dfrac{m\sqrt{(1-A^2\ell^2X)^5}}{4\pi\ell^2\omega^3\alpha^3}$ 'non-hydrodynamic correction' and $\xi = \dfrac{\ell^2}{8\pi\sqrt{3}} = \sqrt{\dfrac{3}{2}}\dfrac{1}{2\pi}k^{1/2}N^{3/2}$

- The quantities depends on the conformal representative $\omega(x)$!

Hubeny, Marolf, Rangamani 2010

- In 2+1, we can play the same game but with a subtle difference
- We introduce $z = x - y$

$$ds^2 = \frac{1}{A^2(x-y)^2}\left(-P(y)dt^2 + \frac{1}{P(y)}dy^2 + \frac{1}{Q(x)}dx^2\right)$$

$$\downarrow$$

$$ds^2 = \frac{1}{A^2 z^2}\left(-Pdt^2 + \frac{(dx^2 + dz^2 - 2dxdz)}{P} + \frac{dx^2}{Q}\right)$$

- Now the conf. boundary is located at $z = 0$
- We can cast the metric into the ADM form where the spacetime coordinates are $x^\mu = (z, \tau, x)$ and induced coordinates on the hypersurfaces are $x^i = (t, x)$
- $$ds^2 = N^2 dz^2 + h_{ij}(dx^i + N^i dz)(dx^j + N^j dz)$$

$$N^2 = \frac{1}{\Omega^2(P+Q)}\,, \qquad N^i = \left(0, -\frac{Q}{P+Q}\right)\,, \qquad h_{ij} = \mathrm{diag}\left(-\frac{P}{\Omega^2}, \frac{Q+P}{\Omega^2 PQ}\right)$$

- The holographic stress tensor $\langle T_{ij}\rangle = \lim_{z\to 0} \frac{1}{8\pi G}\left(K_{ij} - K h_{ij} - \frac{1}{\ell} h_{ij}\right)$
- $\langle T^\tau_\tau\rangle = \frac{\ell A^2 \epsilon}{16\pi G}\left(1 + \epsilon A^2 \ell^2 (3x^2 - 2)\right)$, $\langle T^x_x\rangle = -\frac{\ell A^2\epsilon}{16\pi G}\left(1 - \epsilon A^2 \ell^2 x^2\right)$
- Matches with GAH, Gregory, Scoins 22' for a particular choice of $\omega(x)$
- Full boundary metric and stress tensor.
- Trace anomaly $\langle T^i{}_i\rangle = \frac{c}{24\pi} R[g_{(0)}]$, with $c = 3\ell/2G$ Brown, Henneaux 86

- Fluid/gravity interpretation? Yes, of course.

$$\langle T^i_j\rangle = \begin{pmatrix} -\rho & 0 \\ 0 & \Pi + \frac{\rho}{2} \end{pmatrix}$$

- With $\rho = -\dfrac{A^2\ell(\epsilon + A^2\ell^2(3x^2-2))}{16\pi G}$, $\Pi = \dfrac{A^2\ell(-\epsilon + A^2\ell^2(5x^2-2))}{32\pi G}$.

- We can again use the same trick as in 3+1, $\langle T_{ij}\rangle = \Delta(\xi)\Big(2u_iu_j + g_{(0)ij}\Big) + \Sigma_{ij}$

- Here $\Delta = \dfrac{A^2\ell(\epsilon - A^2x^2\ell^2)}{16\pi G}$, $\Sigma_{ij} = \dfrac{1+\epsilon A^2\ell^2(x^2-1)}{16\pi G A^4\ell} R[g_{(0)}]\delta^\tau_i\delta^\tau_j$

- Same type of behaviour. ‘Non-hydrodynamic’ corrections due to acceleration.

- What does it mean from a real hydrodynamic perspective?

- Euclidean action? $I = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}\,(R-2\Lambda) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\sqrt{-h}K - \frac{1}{\ell}\int_{\partial\mathcal{M}} d^2x\sqrt{-h}$
- On-shell

$$I_{\mathrm{E}} = M + \underbrace{\frac{1}{8\pi G\ell^2 z_h^2\sqrt{\epsilon}}\operatorname{arctanh}\left(\frac{\mathrm{x}}{\sqrt{\mathrm{x}^2-1}}\right)}_{\downarrow\ -TS} + \underbrace{\frac{2\mathrm{A}\sqrt{\epsilon(\mathrm{x}^2-1)}}{\delta}}_{\downarrow\ \text{New divergent contribution due to the wall}}$$

Concluding remarks

- We have constructed a broad family of solutions in 2+1 dimensions resembling the four-dimensional C-metric, showing that the set of possible geometries is much richer than previously acknowledged in the literature.

- Class I_C has no analogous object in higher-dimensions. Deeper understanding is needed.

- Holographic implications still unclear.

- Hydrodynamic expansion

- Proper renormalisation?

- SUGRA construction

Ferrero, Gauntlett, Perez Ipiña,Martelli, Sparks x2 2020
GAH, Donos [In progress]

Questions?

Thanks!